Intuitive Gyroscope Lecture

I have given this lecture fragment several times to first year engineering students after giving the traditional cross product explanation of why a gyroscope suspended at one of its axle ends falls around rather than falling down:

$$\frac{d}{dt}\vec{\alpha} = \vec{\tau} = \vec{r} \times \vec{f}$$

Here:
$\vec{\alpha} = \text{anglar momentum}$, $\vec{\tau} = \text{torque}$, $\vec{r} = \text{lever arm}$, $\vec{f} = \text{force on end of lever}$

This explanation can be made more intuitive.
The above explanation is easily believable if $\vec{\alpha}$ and $\vec{\tau}$ are parallel and we don't need to think of vectors and the right-hand rule for cross products.

In a way, this topic is a good demonstration of the magic of mathematics. An idea that is easy to understand in a simple context can be extended to a realm where intuition fails us.

However, the behavior of a gyroscope can be made intuitive with a series of demonstration materials.

Improved intuition of momentum can also help us understand other devices such as the tuning fork style orientation detector in a smart phone.

Demonstration Materials
1. gyroscope with solid wheel
2. physical optical illusion prop
3. ball on string
4. rod, dumbell, or twiler's baton
5. Gyro-Jack (to be explained later)

Outline of Lecture
1. Demonstrate surprising gyroscope behavior
2. Explain an optical illusion in another context
3. Talk about the plane of rotation of a ball on a string
4. Talk about the cone motion of a conical pendulum
5. Talk about the “hourglass” motion of a rotating rod
6. Describe the game of tether ball and the strategy of changing the plane of rotation
7. Talk about a collection of co-axial co-planer balls on a string that gets perturbed
8. Talk about masses on the arms of a gyro-jack getting perturbed by force on the axle
9. Talk about a wheel composed of infinitesimal masses
10. Explain this as an illusion caused by lack of attention to dynamics

1. Demonstrate surprising gyroscope behavior

This is easy. The students know the math. The gyroscope falls around rather than falling down. Have fun speculating on the possibility of anti-gravity.

2. Explain an optical illusion in another context

This is a warm-up explanation to prepare them to look for something else. Explain that something unexpected is happening with the gyroscope. Maybe they are missing something.

At this point I use a center of gravity illusion that is easy to understand. I use a ceramic model of a flying eagle that balances on its beak because of weighted wing tips ahead of its beak. They can easily understand how they are fooled by the preconception of the weight distribution of an eagle.

3. Talk about the plane of rotation of a ball on a string

Swing the ball on a string around your head (radius 2 m) quickly enough that your hand and the trajectory of the ball is in an almost single plane. Try slightly different planes. Note that if you poke the ball in a particular direction, the axis of rotation will have a corresponding change.

4. Talk about the cone motion of a conical pendulum

Use the ball on a string as a conical pendulum. Note the role of gravity keeping its conical shape.

5. Talk about the “hourglass” motion of a rotating rod

For this, you might like to have a ball on each end of the rod. Holding the rod in the middle, make it move so that each end moves in a circle in its own plane. This is like twirling a baton. It helps to imagine an hourglass shape when imagining the motion.

Explain that you need to continuously apply torque to maintain the “hourglass” motion. Explain that if you release the rod, the “hourglass” motion will collapse into a single plane.

6. Describe the game of tetherball and the strategy of changing the plane of rotation

This is meant to continue focusing their attention on the plane of rotation and the effect of perturbations of trajectory on the plane of rotation.

https://en.wikipedia.org/wiki/Tetherball

The game of tetherball can be played with a volleyball on the end of a string tied to the top of an eight foot pole. Two players are restricted to opposite sides of the pole and try to wrap the tether in their own direction by striking the ball.

The basic strategy is to cause the ball to rotate in a tilted plane over the head of your opponent in a way that you can continue hitting it. If your opponent has a tilt advantage, you might recover your advantage by changing the plane of rotation with a small knock.

7. Talk about a collection of co-axial co-planer balls on a string that gets perturbed
At this point, it is clear to students that a ball on a string will circulate in a fixed plane if the only force on the ball is the centripetal force towards the center of rotation.

The plane of rotation will change if a small impulse parallel to the axis of rotation is applied to the ball. This change in plane of rotation is determined by the change in trajectory of the ball. After the impulse the new axis of rotation will be in a plane perpendicular to the new trajectory.

We can imagine a floppy wheel constructed of many balls on strings rotating in the same plane. We can apply the same impulse to each ball as it passes a chosen position of rotation. They will then have a new shared plane of rotation according to the new trajectories. It is easy to predict the change in direction of the axis of rotation.

8. Talk about masses on the arms of a gyro-jack getting perturbed by force on the axle
Now we can introduce the issue of a rigid axle. The prop I use is a supersized jack from the game of jacks (https://en.wikipedia.org/wiki/Knucklebones).

You can construct a supersized jack (gyro-jack) from tinker toys. Or, you can use a small one from a toy set, because most people know what it looks like, even in a large auditorium.

The point is, that there are four balls on rigid spokes attached to a rigid axle. The balls on the spokes can receive impulses like the balls on a string. As a result, the plane of rotation will change and carry the axis of rotation with it.

Another way to apply impulses to the rotating balls is to perpendicularly tap on the end of the axle. The rigid connection to the spokes transfers the impulses to the balls. And so, the axis of rotation moves perpendicularly to the applied impulse.

Now you can imagine gravity applying a continuum of impulses downward on the end of the axle, creating precession!

9. Talk about a wheel composed of infinitesimal masses
It is now a short leap of intuition to go from a gyro-jack to a solid wheel and the gyroscope demonstration at the beginning.

In principle, if you use elementary $f = ma$ reasoning, with the right initial conditions, you can wind up with the widely used torque and cross product expressions for precession. However, the math is cumbersome and is hard to relate to physical intuition.

10. Explain this as an illusion caused by lack of attention to dynamics
Students still might not feel the truth. In a case where the gyroscope wheel has no holes (or other non-radial appearance), the wheel rotation is less visible.

Furthermore, with the wheel as one solid piece, it is difficult to perceive the individual trajectories of infinitesimal pieces of wheel and the forces on them. The plane of rotation is an abstract geometrical concept that occupies the same physical space as the wheel. The physical object has more cognitive impact than the abstractions of particles and trajectories. Part of the illusion is from gravity acting on mass that is already in motion.

Conclusion

By now students should be able to stare at a spinning wheel and be aware of the trajectories of the infinitesimal masses and be able to predict precession through intuition alone. I hope your students will share an aha moment with you. We will not continue with more complicated gyroscopic motion and tumbling objects in general.

Questions and comments welcome
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